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TRANSVERSE VIBRATIONS OF BELLOWS EXPANSION JOINTS. PART II: BEAM MODEL DEVELOPMENT AND EXPERIMENTAL VERIFICATION

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A theoretical model is developed for the transverse vibrations of bellows expansion joints. The model is based on Timoshenko beam theory and includes the added mass effect of an internal fluid. An analytical expression for bellows natural frequencies is developed in the form of a Rayleigh quotient and is presented in a way which is suitable for hand calculations. The results for the first four transverse modes are compared with experiments as well as the predictions of the simplified analysis of the Expansion Joint Manufacturers Association (EJMA). While the present analysis agrees well with experiments, the EJMA approach can be substantially in error due to its neglect of rotary inertia and the convolution distortion component of fluid added mass.

1. INTRODUCTION

The flexibility of bellows expansion joints makes them susceptible to vibration excited either by external forces or internal flows. As discussed in Part I of this paper (Jakubauskas & Weaver, 1998), this can lead to premature failure due to fatigue. In order to design against damaging vibrations, it is necessary to have good estimates of the bellows natural frequencies. To this end, the Expansion Joint Manufacturers' Association (EJMA 1980) has presented relatively simple expressions for the axial and transverse natural frequencies of vibration of bellows.

Considering only axial vibrations, Jakubauskas & Weaver (1996) used a finite element analysis of a bellows expansion joint modelled as a shell of revolution containing an ideal fluid. The results showed that, while the EJMA predictions were reasonable for the lowest modes of relatively long bellows, significant errors developed as the bellows became shorter and/or the mode number increased. These errors were primarily due to the neglect in the EJMA model of the effect of convolution shape distortion on fluid added mass. However, it was found that the EJMA model estimation of axial stiffness of the bellows could also lead to some error in the prediction of the natural frequency.

Morishita *et al.* (1989) studied the transverse vibrations of a bellows expansion joint using Timoshenko beam theory and finite element analysis. They found good agreement with experiments, at least in the lowest modes, but found substantial error in the EJMA predictions. The latter was attributed to the neglect of rotary inertia in the EJMA model. However, Morishita *et al.* also used a rather simple estimate of the fluid added mass, neglecting convolution distortion.

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The purpose of the research presented in this paper was to develop a theoretical model which more accurately represents the transverse vibrations of fluid-filled bellows while lending itself to hand calculations. To this end, the bellows were modelled as an equivalent Timoshenko beam containing flowing fluid, and the effect of convolution shape distortion during bending on fluid added mass, as developed in Part I of this paper, was included. While the methodology is sufficiently general that it can be used for double (universal) bellows, the results are presented here only for single bellows. The predictions of the approximate Rayleigh quotient formulae are compared with the "exact" solution and with experimental results for the first four modes of lateral vibration. Experiments were conducted in air as well as quiescent and flowing water, so that the predictions for fluid added mass and flow excited frequencies could be validated.

2. THEORETICAL DEVELOPMENT

2.1. General Equation of Motion

Since the transverse vibration of a bellows is being modelled as a relatively short beam in bending, rotary inertia is considered to be important and Timoshenko beam theory is used [see, for example, Timoshenko *et al.* (1974)]. Additionally, since this research was motivated by bellows vibrations excited by internal fluid flow, the beam is represented as a pipe conveying fluid of density ρ_f, velocity, V, pressure, P and net flow area $A_{\min}$ [see, for example, Paidoussis & Li (1993) or Blevins (1990)]. The general equation of motion is then

$$EI_{\mathrm{eq}}\frac{\partial^4 w}{\partial x^4} + m_{\mathrm{tot}}\frac{\partial^2 w}{\partial t^2} - \left(\rho I + \rho\frac{EI}{Gk'}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\rho^2 I}{Gk'}\frac{\partial^4 w}{\partial t^4} + 2\rho_f A_{\min} V\frac{\partial^2 w}{\partial t\, \partial x} + (P\pi R_m^2 + \rho_f A_{\min} V^2)\frac{\partial^2 w}{\partial x^2} = 0, \quad (1)$$

where EI_{eq} is the effective bending stiffness of the bellows, m_{tot} is the total mass per unit length of the bellows including the added mass, ρI in the third term is the effective rotary inertia of the bellows and contained fluid, the terms containing Gk' account for shear, the fifth term is the Coriolis force, and the sixth term accounts for the pipe curvature effects of pressure and centrifugal forces. Note that the pressure is assumed to be constant over the length of the bellows since the pressure drop due to flow losses will be small in comparison with any mean pressure which is large enough to have an effect on the bellows response. All parameters are defined in the Nomenclature. Significant simplification can be achieved by examining the various terms in this equation in detail.

For short beams in bending, the effect of shear deformation is typically of the same order of magnitude, or even greater, than that of rotary inertia. However, Morishita *et al.* (1989) argued that the effect of shear deformation on the bending of bellows was negligibly small. This was confirmed by calculation for the bellows studied in this research, which showed that the ratio of shear to rotary inertia was of the order 10^{-3} (Jakubauskas 1996). Therefore, the shear terms in equation (1) will be neglected.

The Coriolis force can substantially affect the vibration of a pipe, and even generate a dynamic instability for certain pipe boundary conditions, if the flow velocity is sufficiently high. Calculations showed that, even for a typical bellows configuration with a length three times the mean radius, an extreme water velocity of 10 m/s produced less than 0.5% change in the natural frequency in the first two transverse modes (Jakubauskas 1996). Thus, the Coriolis forces are considered negligible for practical bellows applications.

The last term in equation (1) contains the pressure and centrifugal force terms which can reduce the vibration frequency and ultimately lead to buckling. Jakubauskas (1996) showed that velocities up to 10 m/s produced a reduction in transverse natural frequency of less than 0·4%. On the other hand, the maximum pressure in the bellows permitted by the EJMA Standards (1980) produced a reduction in the first-mode natural frequency of 7·6% for the bellows used for experimental validation in this research. Therefore, the pressure term in equation (1) is retained while the centrifugal force term is dropped.

The equation of motion considered valid for transverse vibrations of typical bellows expansion joints containing flowing fluid is therefore assumed to take the following form:

$$EI_{\mathrm{eq}}\frac{\partial^4 w}{\partial x^4}+m_{\mathrm{tot}}\frac{\partial^2 w}{\partial t^2}-\rho I\frac{\partial^4 w}{\partial x^2\,\partial t^2}+P\pi R_m^2\frac{\partial^2 w}{\partial x^2}=0. \tag{2}$$

Before proceeding, it is necessary to determine the equivalent bending stiffness, EI_{eq}, effective moment of inertia, ρI, and total mass per unit length of the bellows, m_{tot}, in terms of the bellows material and geometric properties.

2.2. Equivalent Bending Stiffness, EI_{eq}

The corrugations of a bellows make it impossible to determine the bending stiffness in the usual manner. For the present analysis, the equivalent bending stiffness is determined in terms of the axial stiffness and geometry of the bellows using concepts from the strength of materials.

For a uniform bar of length l, cross-sectional area A and modulus of elasticity E, the axial stiffness, k_b, is given by [see, for example, Beer & Johnston (1981)]

$$k_b=\frac{AE}{l}. \tag{3}$$

Considering now the axial stiffness, k, of a one-half convolution of a bellows, by analogy to the bar of equation (3), the length is half the convolution pitch p, $l=p/2$, and the equivalent cross-sectional area A_{eq} is

$$A_{\mathrm{eq}}=\frac{kp}{2E}. \tag{4}$$

If it is assumed that the bellows can be represented as a thin circular cylinder of mean radius, R_m, then the radius of gyration, r, is given by

$$r^2=\tfrac{1}{2}R_m^2. \tag{5}$$

Continuing with the analogy of the uniform bar in axial tension and using the relationship between the radius of gyration and the moment of inertia from strength of materials, the equivalent moment of inertia for the bellows, I_{eq}, can be determined by using equations (4) and (5):

$$I_{\mathrm{eq}}=A_{\mathrm{eq}}r^2=\frac{kpR_m^2}{4E}. \tag{6}$$

The equivalent bending stiffness of the bellows can then be written as

$$EI_{eq} = \tfrac{1}{4} k p R_m^2. \tag{7}$$

The axial stiffness of bellows, k, can be determined using the simple expressions given by Gerlach (1969) or EJMA (1980). An explanation of the latter is given by Broyles (1994). However, Jakubauskas & Weaver (1996) showed that these formulae under- and overestimate, respectively, the actual stiffness somewhat. Gerlach (1969) recognized this and recommended the use of measured axial stiffness, if possible, for frequency calculations. Axial stiffness calculations for the particular bellows used in this present research for experimental validation of the theory (Jakubauskas 1996) indicated that Gerlach's formula underestimated the finite element predictions by about 8·5% while the EJMA formula overestimated the stiffness by about 18%. To eliminate this error from the theoretical predictions presented in this paper, the axial stiffness obtained from finite element analysis was used in subsequent calculations.

2.3. Bellows Mass per Unit Length, m_{tot}

The total bellows mass per unit length is the mass of the bellows, m_b, plus the fluid added mass, m_f, determined in Part I of this paper. Referring to the bellows geometry in Figure 1, the mass per unit length of the bellows alone, is given by

$$m_b = \rho_b \frac{2\pi R_m [\pi R_1 + \pi R_2 + 2(h - R_1 - R_2)]t}{2(R_1 + R_2)}, \tag{8}$$

where R_m is the mean radius of the bellows as discussed in Part I of this study, ρ_b is the bellows material density, t is the material thickness, the term in brackets in the numerator is the meridional length of a convolution, and $2(R_1 + \mathrm{R}_2) = p$, the length of the convolution pitch. Thus, equation (8) can be rewritten as

$$m_b = \frac{4\pi R_m}{p}(h + 0{\cdot}285p) t \rho_b. \tag{9}$$

Using the expression for fluid added mass, equation (24) from Part I, the total mass per unit length is

$$m_{tot} = \rho_b \frac{4\pi R_m t}{p}(h + 0{\cdot}285p) + \rho_f \left[\pi \left(R_m - \frac{h}{2} + \frac{2hR_2}{p} \right)^2 + \alpha_{f2k} \mu R_m^3 \right], \tag{10}$$

where α_{f2k} is a function involving integrals of the kth mode shape and its second derivative [equation (16), Part I] and μ is the added mass coefficient determined graphically (figure 6, Part I). The first and second terms in the square brackets account for the rigid-body translation and convolution distortion components of fluid added mass, respectively.

2.4. Effective Rotary Inertia of Bellows and Contained Fluid, ρI

The rotary inertia of bellows can be considered as consisting of three parts:

$$\rho I = \rho_b I_b + \rho_f I_{f1} + \rho_f I_{f2}, \tag{11}$$

where ρ_b and ρ_f are the bellows and fluid density, respectively, and I_b, I_{f1} and I_{f2} are the effective moments of inertia of the bellows, the fluid trapped between the bellows convolutions, and the fluid in the central portion of the bellows, respectively. Except for very viscous fluids, I_{f2} is assumed to be negligible [see, for example Paidoussis *et al.* (1986)].

By analogy to an equivalent thin walled pipe, the equivalent cross-sectional area, A_b, can be written

$$A_b = 2\pi R_m t_b, \tag{12}$$

where t_b is the equivalent wall thickness determined from the bellows geometry. Referring to the geometry in Figure 1,

$$t_b = \frac{[\pi R_1 + \pi R_2 + 2(h - R_1 - R_2)]}{p} t. \tag{13}$$

Using the relationship from strength of materials relating the moment of inertia, the cross-sectional area and the radius of gyration, equations (5) and (6), with equations (12) and (13), the effective moment of inertia of the bellows, I_b, can be determined in terms of the bellows geometry.

$$I_b = \frac{\pi R_m^3 t[\pi R_1 + \pi R_2 + 2(h - R_1 - R_2)]}{2(R_1 + R_2)}. \tag{14}$$

A similar methodology can be used to find the effective moment of inertia of the fluid trapped between the bellows convolution. The mean cross-sectional area of the trapped fluid, A_{f1}, is taken as the volume of trapped fluid divided by the convolution pitch, p,

$$A_{f1} = \frac{2\pi R_m h(2R_2 - t)}{2(R_1 + R_2)} \tag{15}$$

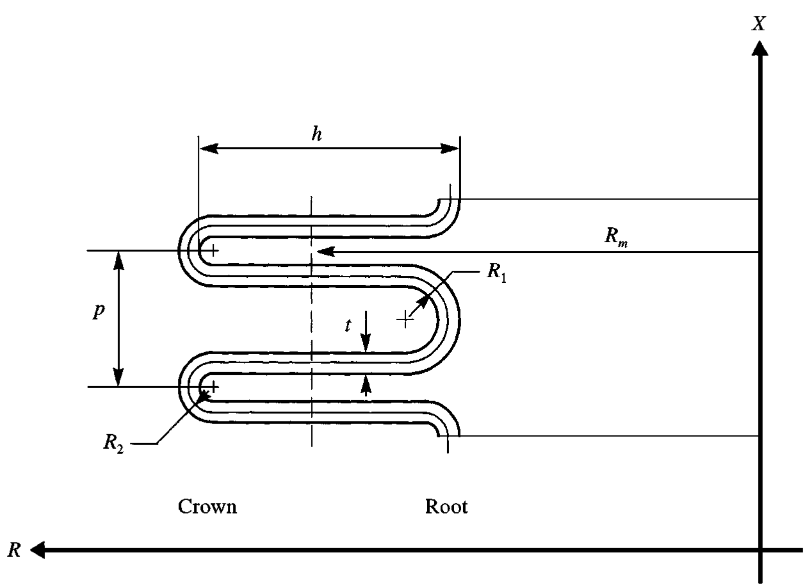

Figure 1. Details of bellows geometry.

Again using equations (5) and (6) from strength of materials, the moment of inertia of the fluid trapped between convolutions is given by

$$I_{f1} = \frac{\pi R_m^3 h(2R_2 - t)}{2(R_1 + R_2)}. \tag{16}$$

The effective rotary inertia of the bellows and contained fluid can now be determined using equations (14) and (16) in equation (11), which becomes after simplification using the bellows geometry

$$\rho I = \pi R_m^3 \left[\rho_b t \left(\frac{2h}{p} + 0{\cdot}571 \right) + \rho_f \frac{h}{p}(2R_2 - t) \right]. \tag{17}$$

2.5. Approximate Solution of the Equation of Motion

The equation of motion for the bellows, equation (2), can be solved using the boundary conditions for a clamped–clamped beam,

$$w(0) = \frac{\mathrm{d}}{\mathrm{d}x} w(0) = w(l) = \frac{\mathrm{d}}{\mathrm{d}x} \mathrm{w}(l) = 0. \tag{18}$$

The assumption of clamped boundary conditions is quite reasonable since the bellows end flanges and connecting pipe are typically much stiffer than the bellows. While the equation of motion (2) subject to boundary conditions, equation (18), can be solved exactly, an approximate solution in the form of a Rayleigh quotient will be adopted here in order to obtain an analytical expression for the bellows natural frequencies. To this end, the solution is assumed to take the form

$$w(x, t) = X(x) \sin \omega t. \tag{19}$$

Substitution of equation (19) into equation (2) yields the ordinary differential equation

$$EI_{\mathrm{eq}} \frac{\mathrm{d}^4 X}{\mathrm{d}x^4} + P\pi R_m^2 \frac{d^2 X}{\mathrm{d}x^2} + \rho I \omega^2 \frac{\mathrm{d}^2 X}{\mathrm{d}x^2} - \omega^2 m_{\mathrm{tot}} X = 0. \tag{20}$$

Multiplying equation (20) by X, integrating by parts and using boundary conditions (18), gives an equation for the natural frequency of the kth mode, ω_k:

$$\omega_k^2 = \frac{EI_{\mathrm{eq}} \int_0^l (X_k'')^2 \, \mathrm{d}x - P\pi R_m^2 \int_0^l (X_k')^2 \, \mathrm{d}x}{m_{\mathrm{tot}} \int_0^l X_k^2 \, \mathrm{d}x + \rho I \int_0^l (X_k')^2 \, \mathrm{d}x}, \tag{21}$$

where a "prime" refers to differentiation with respect to x. Equation (21) can be further simplified by introducing the dimensionless coordinate $\xi = x/l$, using the relationship between equivalent beam stiffness [equation (7)] and converting from circular frequency to cycles per second, $\omega_k = 2\pi f_k$,

$$f_k = \frac{R_m}{4\pi l^2} A_{1k} \left(\frac{kp - 4\pi l^2 P A_{2k}}{m_{\mathrm{tot}} + (\rho I / l^2) A_{4k}} \right)^{1/2}, \tag{22}$$

where

$$A_{1k} = \left[\frac{\int_0^1 (X_k'')^2 \, d\xi}{\int_0^1 X_k^2 \, d\xi} \right]^{1/2}, \quad A_{2k} = \frac{\int_0^1 (X_k')^2 \, d\xi}{\int_0^1 (X_k'')^2 \, d\xi}, \quad A_{4k} = \frac{\int_0^1 (X_k')^2 \, d\xi}{\int_0^1 X_k^2 \, d\xi}. \tag{23}$$

The mode shape functions X_k in equations (23) are understood to be written in terms of the dimensionless coordinate ξ, i.e. $X_k(\xi)$. Thus, once the mode shape function satisfying boundary conditions given by equation (18) are known, the constants A_{ik} of equations (23) can be determined once and for all. A good approximation for the functions X_k can be obtained from the well-known solution of the Bernoulli–Euler equation for a clamped–clamped beam [see, for example, Timoshenko *et al.* (1974)]. Using these beam functions as appropriate mode shape functions for the first four modes in equations (23) generates the solution for the A_{ik} constants given in Table 1.

It should be noted that the coefficient of the convolution distortion component of fluid added mass, α_{f2k}, in equation (10), which was derived in Part I of this paper, also contains the coefficient A_{1k}. This coefficient is expressed in the form [equation (25) of Part I]

$$\alpha_{f2k} = \frac{0{\cdot}066}{l^4} p \left(R_m - \frac{h}{2} \right)^2 A_{1k}^2. \tag{24}$$

2.6. Bellows Stability and Frequency Comparison with Exact Solution

It can be seen from equation (22) that the bellows will become unstable if the term under the square root becomes negative, i.e. if the pressure P becomes sufficiently large. The stability condition for the critical pressure, P_{cr}, in the first mode is therefore

$$P_{cr} = \frac{kp}{4\pi l^2 A_{21}} = 3{\cdot}238 \frac{kp}{l^2} \tag{25}$$

This is quite close to the critical pressure given by EJMA (1980),

$$P_{cr}(\mathrm{EJMA}) = \frac{\pi kp}{l^2}. \tag{26}$$

It appears that the difference is due to the EJMA assumption that the mode shape is a sine function, i.e. $A_{21} = 1/(4\pi^2)$. For design purposes, EJMA (1980) recommends a maximum internal pressure $P_{cr}/6{\cdot}666 \geq P$.

Table 1

Coefficients A_{ik} for first four modes of a single bellows

Mode number k	1	2	3	4
A_{ik}	22·37	61·67	120·9	199·9
A_{2k}	0·02458	0·01211	0·00677	0·00374
A_{4k}	12·30	46·05	98·91	149·4

In order to evaluate the performance of the approximate solution for bellows natural frequencies given by equation (22), the exact solution of equation (20) subject to boundary conditions (18) was computed for the particular bellows used in the experiments described in the following. For the maximum pressure permitted by EJMA (1980) [equation (26)], the approximate frequency prediction was within 0·6% of the exact solution for the first three modes. (A comparison was not made for the fourth mode as this seemed unnecessary.) This excellent agreement is not unexpected, since the error in frequency predictions of the Rayleigh quotient are always much less than the error in the approximate mode shape functions used, and the mode functions used for the calculations are considered to be quite good. This comparison is thought to be important, since now any differences observed between the frequency predictions of equation (22) and experiments can be attributed to theoretical model approximations rather than errors in the Rayleigh quotient approximation.

3. EXPERIMENTAL VERIFICATION

Experiments were conducted using a stainless-steel bellows with the following parameters: $R_m = 0{\cdot}0842$ m, $h = 0{\cdot}0157$ m, $R_1 = 0{\cdot}00353$ m, $R_2 = 0{\cdot}00248$ m, $l = 0{\cdot}1555$ m, $t = 0{\cdot}368$ mm, $E = 2{\cdot}07 \times 10^{11}$ Pa, $\nu = 0{\cdot}3$, $\rho_b = 7860$ kg/m^3. Finite element calculations for the axial stiffness of these bellows gave a value of $k = 1.126$ MN/m per one half-convolution. The measured weight of the bellows without flanges was found to be 4·631 kg/m. Experiments were carried out with a special fixture which permitted determination of the bellows natural frequencies with static air and water at atmospheric and higher pressures. The bellows were then inserted into a water tunnel loop to study flow excitation. This was considered important since the primary motivation for the research was to develop a relatively simple and accurate theoretical model to predict the flow-excited resonant frequencies of bellows. It has been assumed that the fluid flow does not alter the bellows natural frequencies.

3.1. Experiments with Static Air and Water

A special fixture, shown in Figure 2, was designed for these experiments so that the fixture would not influence the bellows dynamics, provide ideal fixed boundary conditions and permit pressurization. The end plates, ① and ③, were made from aluminium with four steel spacers, ②, and tie bolts, ④. This assembly had a fundamental frequency of 2560 Hz, more than 20 times the lowest bellows axial frequency in air (124 Hz) and the lowest transverse bellows frequency when filled with water (112 Hz). Even for the highest mode of interest in this study, the fourth transverse mode in air (about 600 Hz), the fundamental fixture mode was more than 4 times greater. The valves, marked ⑤ and ⑧ in Figure 2, were used for filling the bellows with water and adjusting the pressure in the bellows determined using the pressure gauge, ⑦. The stainless-steel bellows, ⑨, had heavy flanges which were bolted to the end plates using eight bolts, ⑥.

The bellows frequencies were obtained by measuring the bending strains near the convolution crown using two small strain gauges, also shown in Figure 2. These gauges were located precisely 180° apart on the convolution so that, depending on their wiring in the bridge, they could be used to separate axial from transverse vibration modes. The frequency spectra were obtained using a Fourier analyser in the transient capture mode; two typical shock excitation spectra are shown in Figure 3, along with the strain gauge

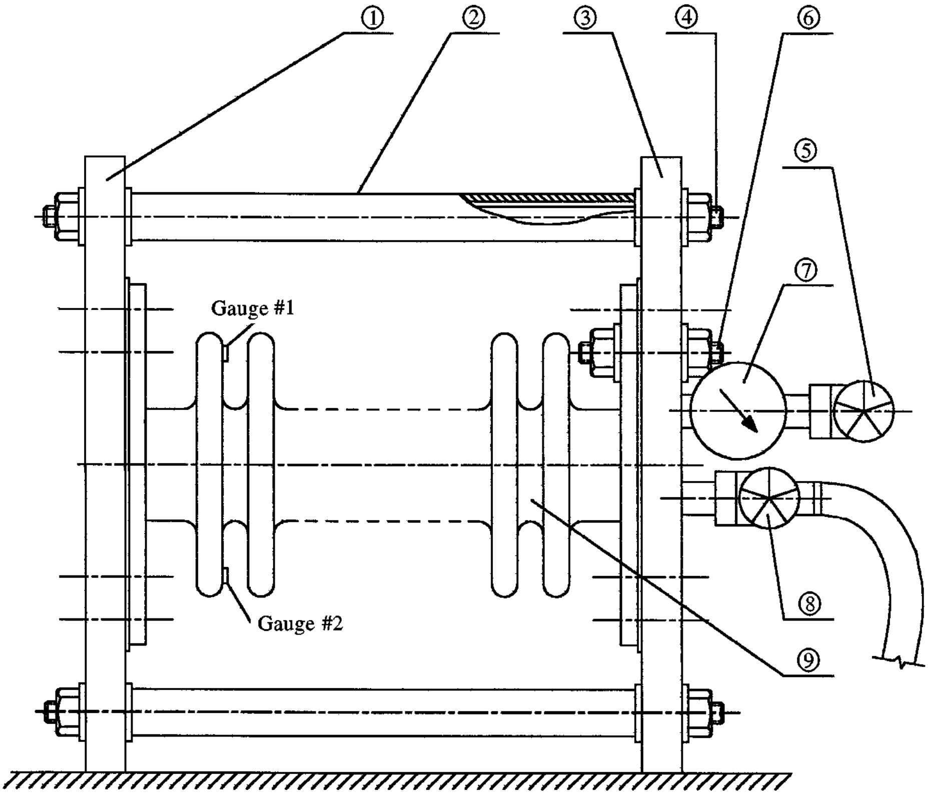

Figure 2. Experimental fixture for vibration testing.

bridge arrangement. Figure 3(a) shows the clear response peaks of the first seven axial modes of the bellows with internal air at atmospheric pressure. Figure 3(b) gives the first four transverse modes for the bellows under the same conditions. The significant results of all such tests are summarized in Table 2 and compared with the theoretical predictions.

Examination of the comparisons in Table 2 reveals some interesting results. First, the theoretical predictions for all cases are quite good, with excellent agreement for the first mode and a maximum error of 5% for the fourth mode in water. Second, fluid-added mass substantially reduces the bellows frequencies (order of 40%) and this is predicted well by the theory. Finally, the effect of pressurization to 200 kPa is to lower the fundamental frequency of the bellows in air by about 7% and this effect diminishes with increasing mode number (about 1·7% for the fourth mode). Again, the theory predicts this behaviour very well. The effect of pressurization of the bellows with water is essentially the same in all aspects so the results have not been shown.

Having demonstrated the validity of the present theoretical model, it is useful to examine the degree to which it represents an improvement over the existing model of EJMA (1980). To this end, calculations were carried out for the cases presented above using Bernoulli–Euler theory with and without the contribution of convolution distortion on added mass and using the simplified approach of the EJMS Standard (1980). The results are summarized in Table 3.

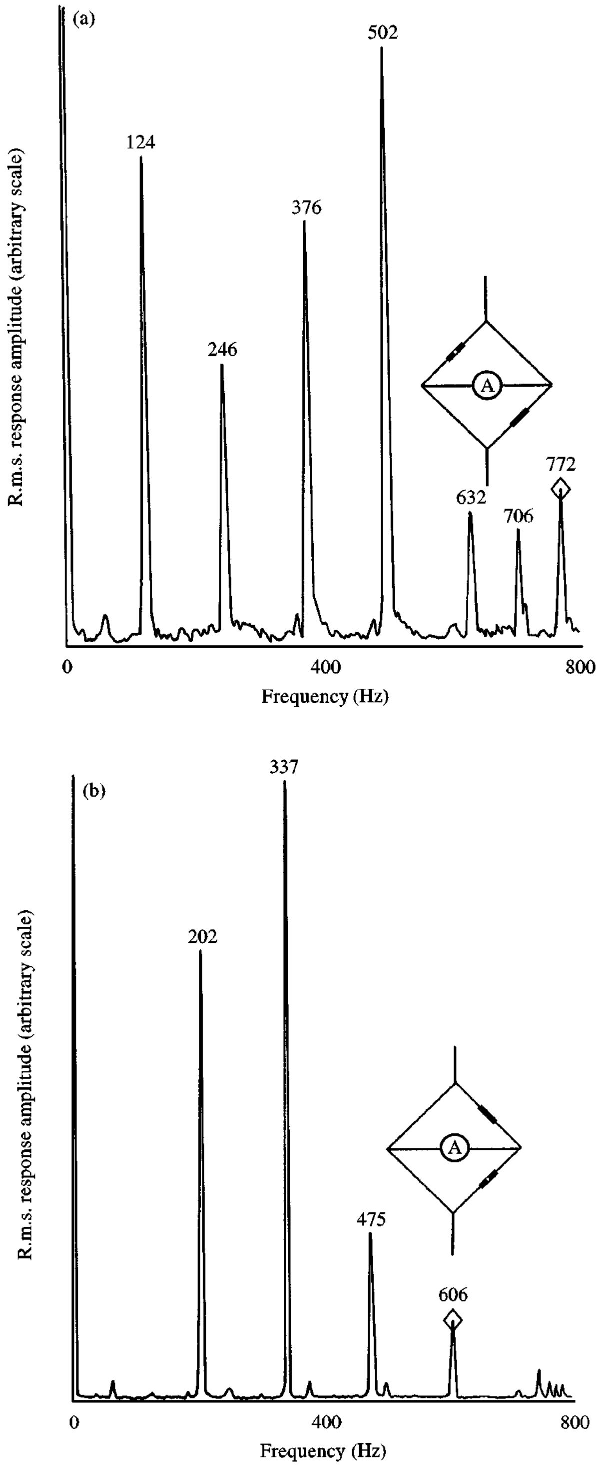

Figure 3. Typical vibration response spectra: (a) axial modes; (b) transverse modes.

TABLE 2
Summary of experimental results (exp.) and comparison with theoretical predictions (theo.)

Mode	Air ($P=0$) Frequency (Hz)			Air ($P = 200$ kPa) Frequency (Hz)			Water ($P=0$) Frequency (Hz)		
	Exp.	Theo.	(% error)	Exp.	Theo.	(% error)	Exp.	Theo.	(% error)
1	202	199	(1·5)	188	187	(0·5)	112	111	(0·9)
2	337	329	(2·4)	328	320	(2·4)	210	208	(1·0)
3	475	455	(4·2)	466	449	(3.6)	289	286	(1·0)
4	606	579	(4·5)	596	574	(3·7)	363	345	(5·0)

TABLE 3
Comparison Bernoulli–Euler beam theory and EJMA model with experiments for air and water

Mode	Frequency (Hz) Air ($P=0$)			Frequency (Hz) Water ($P=0$)			
	Exp.	B-Euler	EJMA	Exp.	B-Euler incl. m_{f2}	B-Euler w/o m_{f2}	EJMA
1	202	344	345	112	137	141	140
2	337	919	923	210	323	388	386
3	475	1792	1810	289	461	761	753
4	606	2977	2992	363	515	1258	1252

The comparisons in Table 3 are very revealing. Since the theoretical calculations using Bernoulli–Euler beam theory agree so well with those of EJMA, it appears that the latter must be based on such an analysis. The results in air show that the neglect of rotary inertia produces a substantial overestimate of the bellows transverse vibration frequencies and that this trend increases with mode number. While still substantially in error, the EJMA estimates for transverse frequencies in water are better than those in air, especially for the first mode. This results from the fact that the rigid-body component of fluid added mass, included in the EJMA approach, dominates rotary inertia and the convolution distortion component of fluid-added mass, m_{f2} in the first mode. As the mode number increases, the convolution distortion component of added mass becomes more significant and the EJMA estimate deteriorates.

It is concluded that rotary inertia must be included in the analysis of transverse vibrations of bellows and that the convolution distortion component of fluid-added mass, while having a rather small effect on the first transverse mode, must be included to obtain reasonable estimates of the higher transverse frequencies of bellows with internal water. Clearly, the extent of these effects will be dependent on the geometry of a particular bellows. The 13 convolution bellows used in this study are considered to be fairly typical and the observed effects of rotary inertia and convolutional distortion are expected to increase for shorter bellows.

3.2. Flow-induced Vibrations of Bellows

In order to study the flow-induced vibrations of the bellows examined above, they were inserted into a closed-return water tunnel (Jakubauskas 1996). In the first set of experiments, the bellows were placed downstream of a straight section of pipe so that the flow through the bellows would be fully developed with a relatively flat velocity profile. The Reynolds number in the range of flows of interest exceeded 10^5. The flow velocity was measured using a pitot-static probe at the centreline of the pipe. The vibration response was obtained using the strain gauges and the Fourier analyser discussed in the previous section although, in this case, the r.m.s. magnitudes of the vibration amplitude peaks in each mode were obtained from 60 sample averaged spectra.

The vibration response was measured starting with a flow velocity of about 1 m/s and then the velocity was incremented. After waiting for sufficient time for steady-state operating conditions to be achieved, the vibration response was obtained as done previously and the whole process repeated to a flow velocity of about 9 m/s. The results are summarized in Figure 4. It should be noted that the response amplitudes are from bending strain measurements on one of the bellows convolutions. Thus, the r.m.s. response amplitudes in Figure 4 are plotted on an "arbitrary scale" and the height of the various peaks should not be interpreted as being indicative of the relative significance of the corresponding modes.

Figure 4 shows that the bellows response was negligible until the velocity exceeded about 1·5 m/s, at which point resonance in the first axial mode (90 Hz) was induced. This resonance peaked at about 2·4 m/s, and then gave way to resonance in the first transverse mode (112 Hz). The latter reached a peak at about 3 m/s and then gave way to resonance in the second axial mode, 176 Hz. This was followed by successive peaks in the third axial mode, 254 Hz and the third transverse mode, 293 Hz. No evidence of the second transverse mode expected at 210 Hz was observed. It appears that this mode was overwhelmed by the adjacent axial modes.

The succession of self-excited flow-induced resonant modes observed here is essentially the same as reported by Weaver & Ainsworth (1989). No significant vibration is observed up to some critical velocity and then, as the flow velocity is increased further, mode after mode becomes unstable in the sequence of the natural frequencies. Weaver & Ainsworth reported a Strouhal number, St = 0·45, in their study.

$$\mathrm{St} = \frac{fp}{V} = 0{\cdot}45, \tag{27}$$

where f is the bellows natural frequency in Hz, p is the bellows pitch (m) and V is the mean flow velocity (m/s) through the bellows, taken at the resonance peak. The data from the present study give an average value for Strouhal number of 0·444 with a standard deviation of 0·011 which agrees well with the previous results.

The importance of these findings is that resonance in the transverse modes can be excited by internal flow in bellows and that the no-flow natural frequencies provide an excellent estimate of the flow-excited frequencies. This confirms the assumptions that Coriolis and centrifugal forces have a negligible effect on the bellows natural frequencies over the range

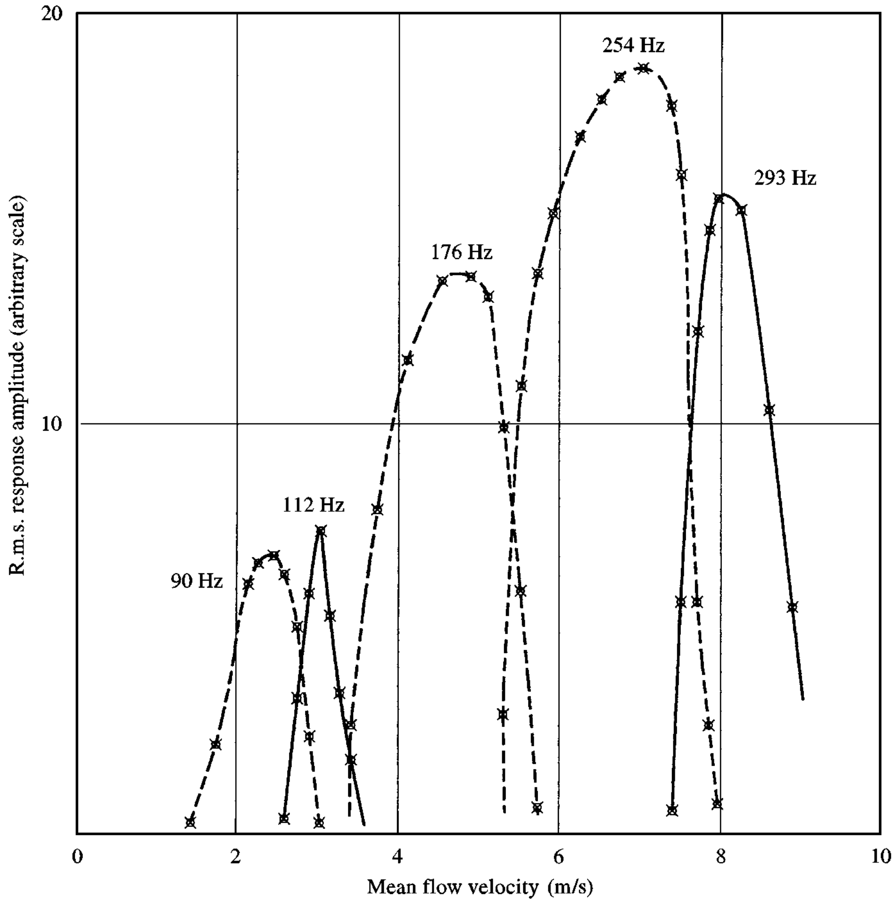

Figure 4. Vibration response as a function of flow velocity (bellows with straight pipe upstream).

of flow velocities of practical interest. These frequencies can be used with a Strouhal number of 0·45 to determine a limiting mean flow velocity for safe operation of the bellows expansion joints. The bellows resonant response amplitudes are sufficiently high that they are clearly visible with the naked eye and, for the range of frequencies in this study, also clearly audible. Continuous operation at resonance will likely produce fatigue failures in a relatively short period of time. This was observed in the experiments of Weaver & Ainsworth (1989).

3.3. Bellows Near an Upstream Elbow

It is commonplace to locate bellows expansion joints immediately downstream of pipe elbows. Weaver & Ainsworth (1989) observed that bellows could be excited to resonance even if only a portion of the circumference of the convolutions was exposed to a sufficiently high flow velocity. It is well known that flow separation at the inside radius of a pipe elbow produces a flow velocity at the downstream outside radius which is substantially higher

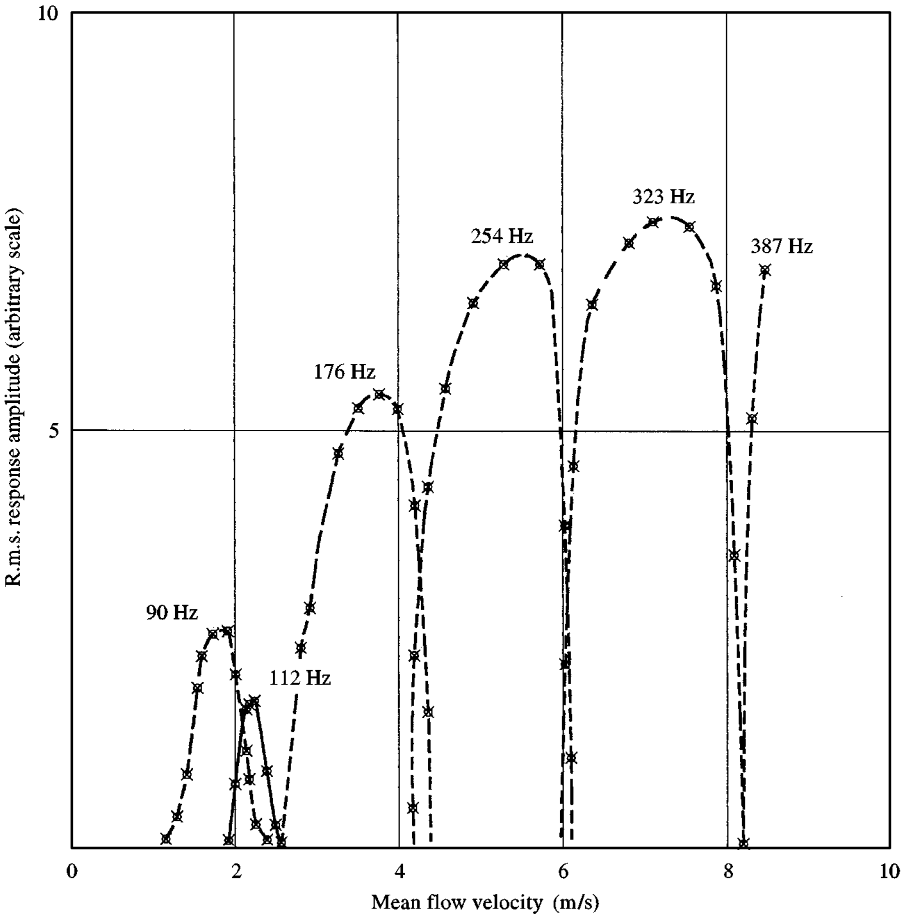

Figure 5. Vibration response as a function of flow velocity (bellows just downstream of a 90% elbow).

than the mean pipe flow velocity. Thus, it is reasonable to expect that, if a bellows is exposed to such a nonuniform flow, then it could be excited to resonance at lower mean flow velocities than bellows exposed to uniform flows. To examine this possibility, the bellows of Section 3.2 were located immediately downstream of a standard 90° radiused elbow in the water tunnel pipeline and the experiments repeated. The results are plotted in Figure 5.

Careful examination of Figure 5 shows that the overall response behaviour of the bellows as a function of flow velocity is essentially the same as observed for uniform flows in Figure 4. The excited frequencies have the same values and sequence, except that the third transverse mode at 293 Hz has dropped out, while the fourth and fifth axial modes have appeared at 323 and 387 Hz, respectively. The most important difference between Figures 4 and 5 is that the resonance peaks all appear at lower mean flow velocities in the latter case. The average Strouhal number, based on mean pipe flow velocity, corresponding to the resonance peaks in Figure 5 is 0.574 with a standard deviation of 0·046. The scatter in these results is clearly greater than for the uniform flow case, as might be expected. However, the important practical implication is that the mean flow velocity required to excite resonance when the bellows are immediately downstream of a 90° elbow is, on average, 29% lower than that required when the bellows are exposed to uniform flow.

4. CONCLUSIONS

A theoretical model for the transverse bending vibrations of bellows has been developed. The model includes the effects of rotary inertia and fluid added mass and is presented in a way that permits hand calculation of natural frequencies. The theoretical predictions were compared with those of the simple model of EJMA (1980) and with experimental results for bellows in air, and in quiescent and flowing water. The principal conclusions are summarized as follows.

1. The excellent agreement between the present theory and experiment tends to validate the assumptions used in modelling the fluid added mass and the bellows as a Timoshenko beam with negligible shear.
2. The EJMA (1980) model substantially overestimated the transverse natural frequencies of the bellows used in this study in both air and water. This is due to the neglect of rotary inertia when added mass effects due to convolution distortion are small, i.e. for bellows in air or for the first transverse mode of relatively long bellows filled with water.
3. The convolution distortion components of fluid added mass is small for the first transverse mode of relatively long bellows, i.e. it lowered this mode frequency about 3% for the bellows studied. However, this fluid-added mass component becomes increasingly important as the bellows length is reduced and/or the transverse mode number is increased. For the bellows used in this study, neglect of the convolution distortion component of added mass in the fourth mode results in an overestimation of the natural frequency by about 140%. Neglect of this added mass component and rotary inertia in the fourth mode of the present bellows in water results in a frequency prediction which is over 240% too high.
4. Fluid flowing through bellows is expected to have a negligible effect on their natural frequencies, at least in most practical applications. For the present bellows, no effect was observed up to a mean water velocity of 10 m/s.
5. The effect of internal pressurization on bellows natural frequencies is relatively small on the first transverse mode and decreases with increasing mode numbers. For the present bellows this effect was about 7% for the first mode and less than 2% for the fourth mode.
6. The presence of a 90° radiused elbow immediately upstream of a bellows can substantially reduce the mean flow velocity required to excite bellows to resonance. For the present bellows, this effect averaged about 29% over all the modes studied.

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APPENDIX: NOMENCLATURE

A_b	bellows equivalent area to a thin pipe
A	cross-sectional area
A_{eq}	equivalent bellows cross-sectional area
A_{min}	net flow area through bellows
A_{f1}	mean area of fluid trapped in bellows convolution
A_{ik}	ith constant for mode k
E	modulus of elasticity
EI	flexural stiffness of beam
EI_{eq}	equivalent bending stiffness of bellows
f	frequency (Hz)
G	shear modulus of elasticity
h	total convolution height
I	second moment of area (moment of inertia)
I_{eq}	bellows equivalent moment of inertia
I_b	effective bellows moment of inertia for rotation
I_{f1}	effective moment of inertia for rotation of trapped fluid
I_{f2}	effective moment of inertia for central fluid in bellows
k	axial stiffness of bellows
k_b	axial stiffness of bar
k'	shear constant
l	length of beam, bar or bellows
m_{tot}	total bellows mass per unit length (including fluid added mass)
m_b	bellows mass per unit length
p	convolution pitch
P	fluid pressure
r	radius of gyration
R_1	convolution root radius
R_2	convolution crown radius
R_m	mean radius of bellows
St	Strouhal number
t	bellows convolution thickness

t_b	bellows equivalent wall thickness referred to a thin-walled pipe of radius R_m
V	mean flow velocity through bellows
w	transverse displacement
X_k	bellows mode shape in kth mode
α_{f2k}	integral function for bellows distortion fluid added mass in kth mode
μ	added mass coefficient
ρ	density
ρ_b	density of bellows material
ρ_f	fluid density
ω	frequency (rad/s)
ω_k	bellows natural frequency, kth mode

Subscript

k	kth mode